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Procedia Engineering 2 (2010) 1317–1325

Procedia Engineering

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Fatigue 2010

Fatigue life prediction of notched components based on a new nonlinear Continuum Damage Mechanics model

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Received 8 March 2010; revised 12 March 2010; accepted 15 March 2010

Abstract

A CDM (continuum damage mechanics) model for damage evaluation is here considered and applied to the study of two different typologies of notched and cylindrical specimens. The model presents some parameter and authors determined them in a previous work testing cylindrical and smooth specimens.
Firstly fatigue characterization was conducted and the SN curves found. In order to evaluate the capability of CDM model to predict the sequence effect and to simulate a more realistic loading condition, tests with various loading blocks were carried on and in particular high-low, low-high and random blocks were applied to the three specimens considered. Model previsions showed good agreement with results for each geometry considered.

Keywords: Continuum damage mechanics; variable amplitude fatigue; load sequence effect.

Nomenclature

D	damage variable
n_i	number of cycles at a given stress amplitude
N_i	number of cycles to failure at a given stress amplitude
σ_{max}	maximum stress
σ_m	mean stress
σ_a	stress amplitude, $\sigma_a = \sigma_{max} - \sigma_m$
M_0, β	coefficients of the damage model
$\alpha(\,)$	function in the damage model
H, a	coefficients of function $\alpha(\,)$
$\langle\,\rangle$	defined as $\langle x\rangle = 0$ if $x < 0$, $\langle x\rangle = x$ if $x > 0$

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doi:10.1016/j.proeng.2010.03.143

σ_u	ultimate strength
σ_y	yield strength
σ_f	fatigue limit
V, W	auxiliary variables used for the calculation of damage D

1. Introduction

Fatigue failure in metallic materials represents a central topic not only in research field, but else in industrial reality. In fact the large part of mechanical components and the most critical ones work with variable loads and their dimensioning is crucial for the integrity of entire structure or for the safety of the workers and the users. An important factor that characterizes fatigue phenomena is the high dispersion of data that results typical in any observation even if the same procedures and conditions are kept and this aspect makes necessary the introduction of a statistical analysis of data. Many different approaches are employed in studying fatigue: the choice of the most appropriate model depends on the criticism of the structural component and on the number of applied cycle (LCF or HCF). Therefore, "damage tolerance" defined as "the ability to resist failure due to the presence of flaws, cracks or other damage for a specified period of unrepaired usage" is applied where a failure should cause an important damage or the overall failure of the structure; while "durability", defined as "the ability to resist cracking, corrosion, deterioration, thermal degradation, delamination, wear and the effects of foreign and domestic object damage for a specified period of time", is used when it is possible to reduce the maintenance or when the considered part can be easily replaced with a new one [1].

It is well known that fatigue damage is progressive and irreversible and it accumulates bringing the material to failure. Some mechanical property changes during the fatigue loading and knowing one of them it is possible to appreciate the damage state of material. Anyway this measure is often difficult to be performed and introduces uncertainty to a phenomenon that already presents a high dispersion of data.

Close to the aforementioned methodologies, an attempt to model and quantify the fatigue damage has been done since the beginning of fatigue studies and without any employment of mechanical measures. The first to enunciate a law was Palmgreen and later Miner gave a precise mathematical formulation in the form [2]:

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \tag{1}$$

Even if this law has many conceptual limits (it is based on assumption of constant work absorption per cycle independently on the loading level), it has been considered as valid (with opportune modification) in international standards and represents the comparison term for the evaluation of other models for damage calculation. Many other formulations and laws were proposed in order to consider the main effects observed in fatigue in metals: effect of load sequence, damage produced with load lower than fatigue limit and non-linearity of damage accumulation [3-7].

Kachanov [8] introduced the concept of continuum damage in describing the mechanical deterioration due to creep and fatigue phenomena and later Chaboche and Lemaitre applied these principles to formulate a non-linear damage evolution equation in the form:

$$\delta D = f(D, \sigma)\delta n \tag{2}$$

that makes the variable D and the load parameters non-separable [9, 10].

This approach has been adopted by many authors [11-16] even if with integrations and modifications. Dattoma et al. [17] proposed a new non-linear uniaxial model based on CDM (Continuum Damage Mechanics) starting from the original framework, and a new formulation for coefficients was proposed and experimentally verified. The model was tested on cylindrical un-notched specimens and the experiments showed good agreement with the formulation else for complex load histories. In this work, the proposed model is applied for evaluation of cumulative fatigue damage in two other different typologies of notched cylindrical specimens. S-N curves were drawn and compared and than employed for damage calculation by means of CDM model.

2. CDM model

The equation proposed by Chaboche [9] for fatigue damage in a uniaxial problem is defined by the following differential equation:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha(\sigma_{\max},\sigma_m)}\left[\frac{\sigma_a}{M_0(1-b\sigma_m)(1-D)}\right]^{\beta}\delta n \qquad (3)$$

where β, M_0, and b depend on material; the exponent α also depends on the loading (σ_{max} , σ_m) which results in non-separability between damage and loading. If the load ratio R=–1 (pure oscillating stress) is considered the starting equation becomes:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha}\left[\frac{\sigma_a}{M_0(1-D)}\right]^{\beta}\delta n \qquad (4)$$

The expression of α has a crucial role in the calculation of damage with CDM and for this considered model it is taken as:

$$\alpha = \begin{cases} 1 & for\ \sigma_a < \sigma_f \\ 1-\dfrac{1}{H}\left\langle\dfrac{\sigma_a-\sigma_f}{\sigma_u-\sigma_a}\right\rangle^{a} & for\ \sigma_a \geq \sigma_f \end{cases} \qquad (5)$$

where a and H are parameters to be experimentally determined.

Integrating from $D = 0$ to $D = 1$, the equation (4) gives the number of cycles to failure for a given load:

$$N_f = \frac{1}{1-\alpha}\frac{1}{1+\beta}\left[\frac{\sigma_a}{M_0}\right]^{-\beta} \qquad (6)$$

that confirms the almost linear relation between Log$S$ and Log$N$.

Integrating the differential equation in a generic instant before failure ($D < 1$ and $n < N_f$) it is possible to express the damage, D as a function on n/N_f , as follows:

$$n = \frac{1}{1-\alpha}\frac{1}{1+\beta}\left[\frac{\sigma_a}{M_0}\right]^{-\beta}\left[1-(1-D)^{1+\beta}\right]^{1-\alpha} \Rightarrow D = 1-\left[1-\left(\frac{n}{N_f}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{1+\beta}} \qquad (7)$$

Considering a load history composed by two blocks (σ_{a1} for n_1 cycles and then σ_{a2} for n_2 that brings a specimen to failure) it is possible to demonstrate that it is:

$$\frac{n_2}{N_{f_2}} = 1-\frac{N_2}{N_{f_2}} \Rightarrow \frac{n_2}{N_{f_2}} = 1-\left(\frac{n_1}{N_{f_1}}\right)^{\frac{1-\alpha_2}{1-\alpha_1}} \qquad (8)$$

In the case of high-low loading ($\sigma_{a1} > \sigma_{a2}$), from equation (5), it follows:

$$\frac{n_1}{N_{f_1}} + \frac{n_2}{N_{f_2}} < 1 \tag{9}$$

This demonstrates the model can correctly predict the effects of loading interaction and in the case of high-low sequence, the damage calculated with Palmgreen-Miner rule (D_{LDR})in correspondence of failure is less than unit; likewise it can be demonstrated that in the case of low-high loading, D_{LDR} is more than unit when failure occurs.

Adopting this approach and introducing an auxiliary variable, V, it is possible to get a simple formula for cumulative damage when more different blocks higher than fatigue limit are applied:

$$\begin{cases} V_i = \left(\dfrac{N_i + n_i}{N_{f_i}} \right)^{\frac{1}{1-\alpha_i}} = \left\{ \left[1 - (1 - D_{i-1})^{1+\beta} \right]^{1-\alpha_i} + \dfrac{n_i}{N_{f_i}} \right\}^{\frac{1}{1-\alpha_i}} \\ D_i = 1 - \left[1 - \left(\dfrac{N_i + n_i}{N_{f_i}} \right)^{\frac{1}{1-\alpha_i}} \right]^{\frac{1}{1+\beta}} = 1 - \left[1 - V_i \right]^{\frac{1}{1+\beta}} \end{cases} \tag{10}$$

When loading blocks lower than σ_f are present, α is equal to 1 and equation (4) becomes:

$$\delta D = \frac{\left[1 - (1 - D)^{1+\beta} \right]}{(1 - D)^{\beta}} \left[\frac{\sigma_a}{M_0} \right]^{\beta} \delta n \tag{11}$$

After integration of (11) it gives:

$$W_i = \left[1 - (1 - D_{i-1})^{1+\beta} \right] e^{\frac{n_i}{N'_i}(1+\beta)} ; N'_i = \left[M_0 / \sigma_{ai} \right]^{\beta} \Rightarrow D_i = 1 - \left[1 - W_i \right]^{\frac{1}{1+\beta}} \tag{12}$$

The exposed formulation is consistent with physical significance of the fatigue damage, in fact:

- fatigue damage is an irreversible process of material degradation and it increases monotonically with the applied cycles:

$$\frac{\partial D}{\partial n} > 0 \tag{13}$$

- larger provoked fatigue damage corresponds to higher applied load:

$$\frac{\partial^2 D}{\partial n \partial \sigma} > 0 \tag{14}$$

3. Experimental setup and characterization

A hardened and tempered steel, 30NiCrMoV12, mostly used for railway axle, has been considered for testing the model. Its mechanical characteristic are reported in Table 1

Table 1. Mechanical properties of steel 30NiCrMoV12.

E [GPa]	σ_y [MPa]	σ_u [MPa]
201,4	*775*	*1035*

Two different shapes for notched specimens were considered (here called "NOT1" and "NOT2" as reported in Figure 1), while un-notched specimens (called "UNNOT") were employed for static and for fatigue test and characterization [17].

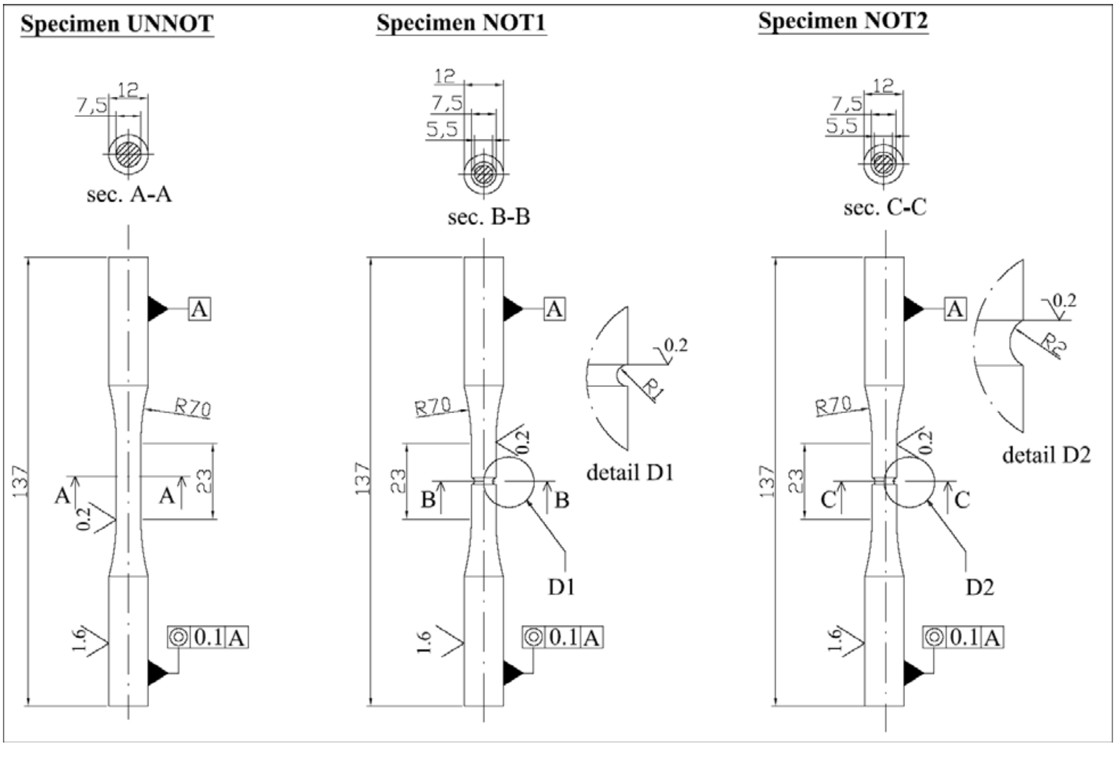


Fig. 1. Geometries of specimens.

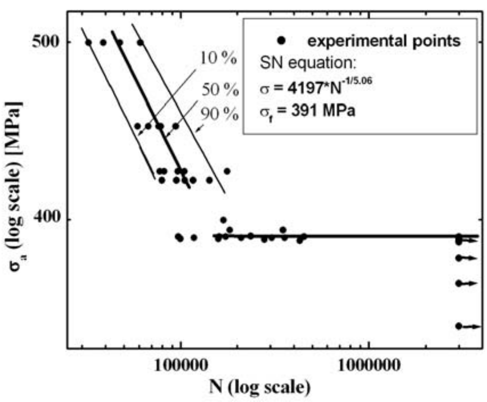
All tests were performed with a servo-hydraulic MTS 810 test machine with load cells of 10 kN and 100 kN with a frequency of 25 Hz in force control mode.

Fatigue characterization has been conducted according to constant stress level and survival method procedures [18]; the SN curves at three probabilities of failure (10%, 50% and 90%) and fatigue limits for the studied specimens were obtained and reported on the diagram in Figures 2 (a), (b) and (c).

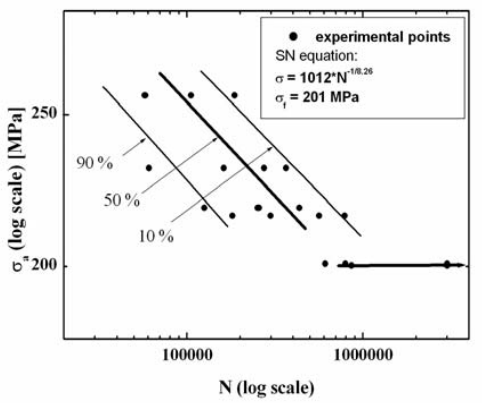
The CDM model parameters, α, β, H and M_0, for this material were experimentally determined by Dattoma et al [17] (see Table 2) and the data relative to SN curves relative at 50% of probability of rupture were here employed.

Table 2. Parameter of the CDM model as calculated by Dattoma et al. [17].

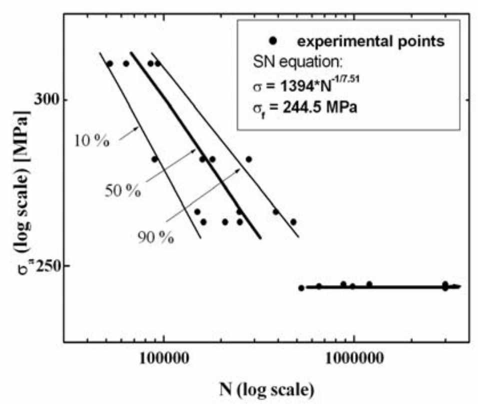
a	β	H	M0
0.434	*0.758*	*0.0801*	*1.54×10^{10}*



(a)



(b)



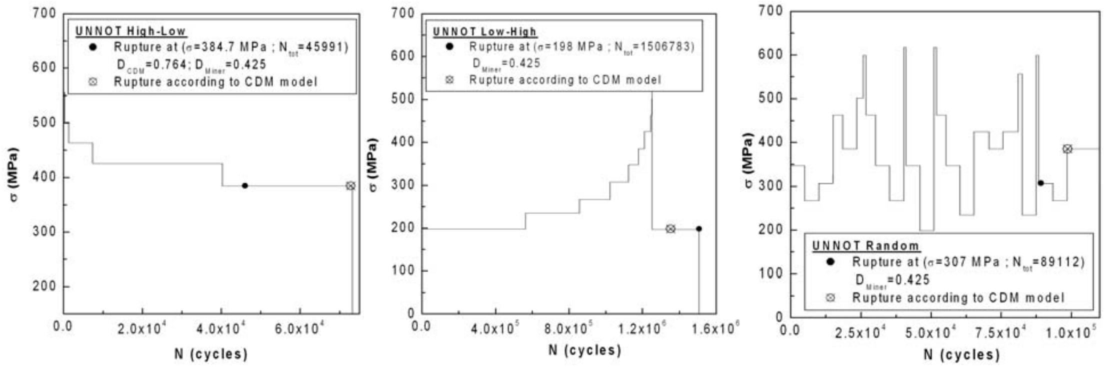
(c)

Fig. 2. Diagrams with S-N curves for the three groups of specimens UNNOT (a), NOT1 (b) and NOT2 (c). The curves relative at three probabilities of rupture (10%, 50%, 90%) are drawn.

4. Experimental results

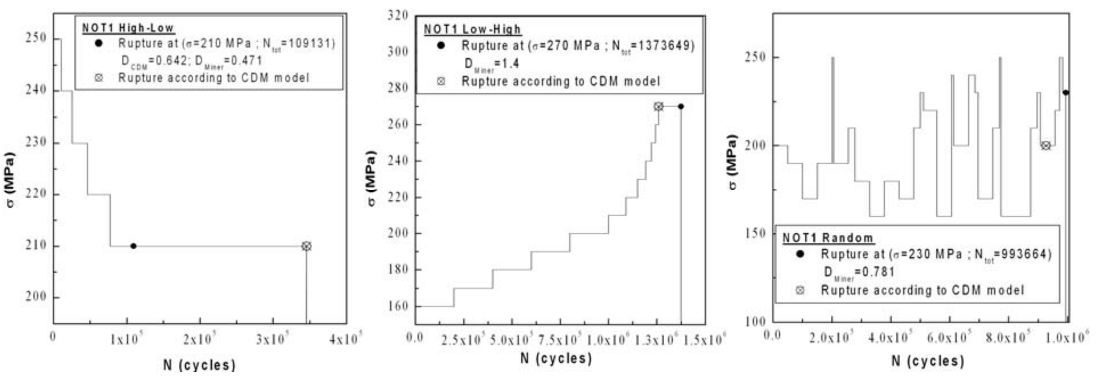
Even if the model theoretically considers the effect of load sequence it is important to estimate the reliability in evaluating damage and especially the number of cycles to failure in a multi-block test. At this aim high-low and low-high loading sequences were imposed for each typology of specimen. Moreover random loading condition was simulated to appreciate the capability of CDM model to predict failure in presence of a load history similar to a real one.

In every test, cumulative damage was calculated with LDR and also with CDM (where possible) to have an immediate comparison.



(a) (b) (c)

Fig. 3. Diagrams representing the application of consecutive blocks of load for UNNOT specimens in three different configurations: (a) high-low, (b) low-high, (c) random.



(a) (b) (c)

Fig. 4. Diagrams representing the application of consecutive blocks of load for NOT1 specimens in three different configurations: (a) high-low, (b) low-high, (c) random.

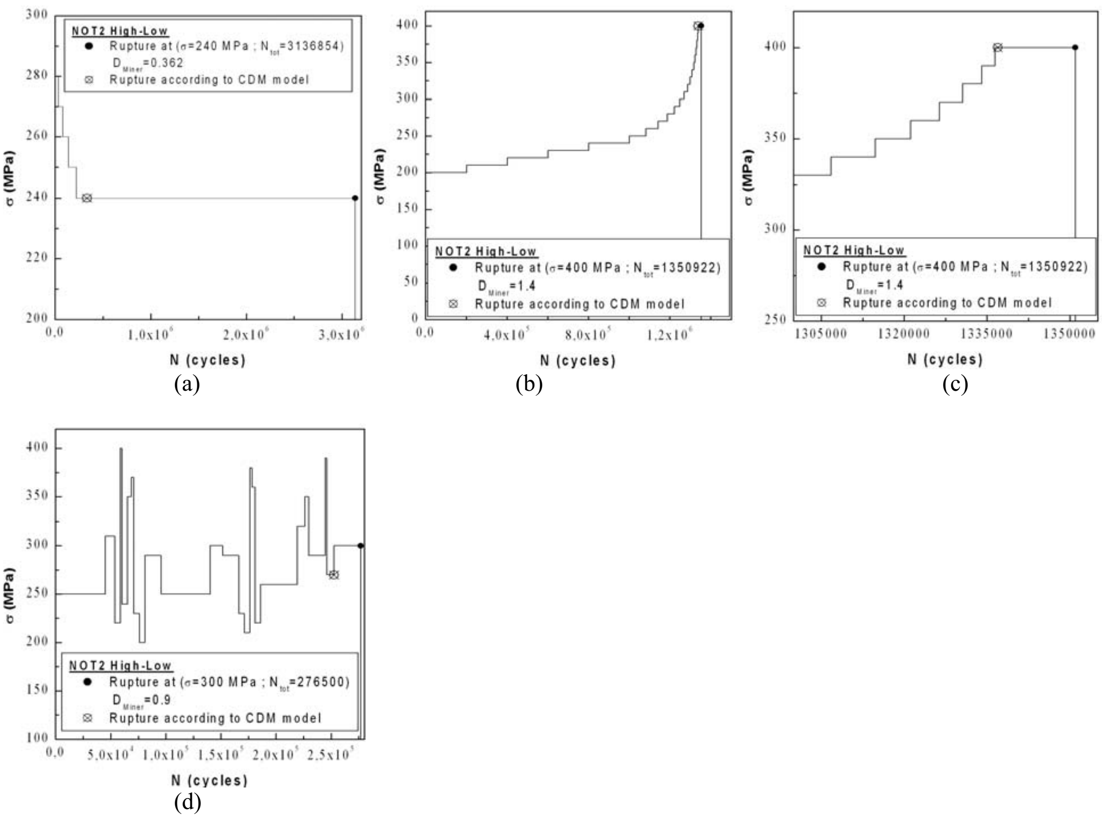


Fig. 5. Diagrams representing the application of consecutive blocks of load for NOT2 specimens in three different configurations: (a) high-low, (b and c) low-high, (d) random; (b) and (c) represents the same test with different scale to highlight the final blocks close to failure.

Results in figures 3, 4, 5 highlight one important feature of CDM model here considered: the capacity to correctly take into account the load sequence effect. This aspect, even if theoretically verified, is here strongly confirmed to be valid not only for a smooth specimen but for else a notched one. It implies that for first immediate evaluation a designer does not need to know how the stress is distributed around the notch and the only tools he needs are: S-N curves, parameters of the model for the considered material. These conditions can be obtained with a relative simple procedure and employing well-established methodologies.

5. Conclusions

In this paper a CDM model proposed by the authors in a previous work is considered. The formulation is coherent with some of the main effect observed when a mechanical part undergoes fatigue loads and particularly important is the load sequence effect.

Three different geometries of specimen (made in 30NiCrMoV12 steel) are considered: the first one is smooth, while the two others present two different notches.

Firstly mechanical characterization was conducted and S-N curves were obtained for the different geometries and tests with variable load were conducted. Cumulative damage was evaluated for every test and the results are encouraging. Therefore a first conclusion is the proposed CDM model works for complex geometry and not only for smooth specimen.

An evident disadvantage of the methodology here proposed lies in the necessity to know the S-N curve for a new considered geometry and for different load conditions (different load gradient or load ratio). It implies the possibility to perform new tests and to introduce some error. Authors are actually working to the elaboration of the proposed model introducing new other parameters in order to employ the same S-N curve (the one referred to smooth geometry) for any geometry or load condition.

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